Assessment Schedule – 2008

Calculus: Integrate functions and use integrals to solve problems (90636)

Evidence Statement

	Achievement Criteria	Q.	Evidence	Code	Judgement	Sufficiency
Achievement	Integrate functions and use integrals to solve problems.	ONE (a)	$-\frac{1}{18}(2-3x)^6+c$	A1	**Do not penalise for omission of constant.**	**Achievement:** **Four** of Code A **including** at least **one** A1 and **one** A2.
		(b)	$\frac{-3}{2x^2}-4\ln\lvert x\rvert+c$ or $-\frac{3}{2x^2}-4\ln\lvert kx\rvert$	A1	Or equivalent. Or equivalent. Accept without | | sign. Log x not accepted.	
		TWO	Using Trapezium rule $\text{Area} = \frac{40}{2}(110+125+2(150+100+75))$ $= 17\ 700\ \text{m}^2$ Right formula using 140 as final value. $\text{Area} = \frac{40}{2}(110 + 140 + 2(150 + 100 + 75))$ $=1800\ \text{m}^2$ No Bracket formula $\text{Area} = \frac{40}{2} \times 110 + 125 + 2(150 + 100 + 75)$ $=2975\ \text{m}^2$ No Bracket Formula with 140 final value. $\text{Area} = \frac{40}{2} \times 110 + 140 + 2(150 + 100 + 75)$ $= 2990\ \text{m}^2$ OR using Simpson's rule $A = \frac{40}{3}(110 + 125 + 4(150 + 75) + 2(100))$ $= 17\ 800\ \text{m}^2$ Right formula using 140 as final value. $A = \frac{40}{3}(110 + 140 + 4(150 + 75) + 2(100))$ $=1800\ \text{m}^2$ No Bracket formula $A = \frac{40}{3} \times 110 + 125 + 4(150 + 75) + 2(100)$ $=2691.67\ \text{m}^2$ No Bracket Formula with 140 final value. $A = \frac{40}{3} \times 110 + 140 + 4(150 + 75) + 2(100)$ $= 2706.67\ \text{m}^2$	A2	Either method accepted. Units not reqd.	

	Achievement Criteria	Q.	Evidence	Code	Judgement	Sufficiency
		THREE	$= \int_5^{100} \left(e^{0.05x} + 250\right) dx$ $= \left[20e^{0.05x} + 250x\right]_5^{100}$ $=27968 - 1276$ $=26\,693\ m^2$	A1 A2	Must show integrated function. Units not reqd. A1 for correct indefinite integral or A2 for correct answer.	**Achievement:** **Four** of Code A **including** at least **one** A1 and **one** A2.
		FOUR	$\frac{dV}{dt} = \frac{50}{(t+1)^2}$ $V = \int \frac{50}{(t+1)^2} dt$ $= \frac{-50}{(t+1)} + 60$ since at $t = 0$, $V = 10$ At $t = 60$, $V = -\frac{50}{61} + 60 = 59.2$ litres	A1 A2	Units not required	
Achievement with Merit	Use advanced integration techniques to find integrals and solve problems.	FIVE	Let $u = x - 1$ then $x = u + 1$ and $du = dx$ $I = \int \frac{3u+11}{\sqrt{u}} du$ $= \int 3\sqrt{u} + 11u^{-0.5} du$ $= 2u^{1.5} + 22u^{0.5} + c$ $= 2(x-1)^{\frac{3}{2}} + 22(x-1)^{\frac{1}{2}} + c$ Using $u = \sqrt{(x-1)}$, $\frac{dy}{dx} = \frac{1}{2\sqrt{(x-1)}}$ $u^2 = x - 1$ $= \frac{1}{2u}$ $x = u^2 + 1$ $du = 2udx$ $3x + 8 = 3u^2 + 2 + 8$ $I = \int \frac{3u^2 + 11}{u} \times 2udu$ $= \int (6u^2 + 22) du$ $= 2u^3 + 22u + c$ $= 2\left(\sqrt{x-1}\right)^3 + 22\sqrt{x-1} + c$	A1 M	Or equivalent.	**Merit:** Achievement **plus** **Two** of Code M **OR** **Three** of Code M.

	Achievement Criteria	Q.	Evidence	Code	Judgement	Sufficiency
		SIX	$v(t) = 3e^{0.2t} + c$ $v(2) = 3e^{0.4} + c = 5$ $c = 0.525$ $v(t) = 3e^{0.2t} + 0.525$ Dist Travelled $= \int_3^4 (3e^{0.2t} + 0.525)dt$ $= \left[15e^{0.2t} + 0.525t + k\right]_3^4$ $= 35.4812 - 28.9054$ $= 6.576$ m	A1 M	Must show integrated function. Units not required.	**Merit:** Achievement **plus** **Two** of Code M **OR** **Three** of Code M.
		SEVEN	$\frac{dV}{dt} = kV$ $\int \frac{dV}{v} = \int k dt$ $\ln\|V\| = kt + c$ $V = V_0 e^{kt}$ $V = 16e^{kt}$ $0.5 = e^{8k}$ $k = \frac{1}{8}\ln(0.5) = -0.0866$ $5 = 16e^{-0.0866t}$ $t = \frac{\ln\frac{5}{16}}{-0.0866}$ Drug will be effective for 13.42 hours.	M	Accept $V = V_0 e^{kt}$ without working. Must evaluate k. Or equivalent. Accept any valid method. Accept minor arithmetic error.	

	Achievement Criteria	Q.	Evidence	Code	Judgement	Sufficiency
Achievement with Excellence	Solve more complex integration problem(s).	EIGHT	Equation of line: $y = \frac{h}{b-a}(x-a)$ $x = \frac{(b-a)y}{h} + a$ Volume $= \pi \int_0^h x^2 \mathrm{d}y$ $= \pi \int_0^h \left(\frac{(b-a)y}{h} + a \right)^2 \mathrm{d}y$ $= \pi \left[\frac{(b-a)^2 y^3}{3h^2} + a\frac{(b-a)y^2}{h} + a^2 y \right]_0^h$ $= \pi h \left(\frac{(b-a)^2}{3} + ab \right)$	M E	 Or equivalent.	**Excellence:** Merit **plus** Code E **OR** Two of Code E.
		NINE	$\sin kx = \sin^2 kx$ $\sin x(\sin kx - 1) = 0$ $x = 0 \text{ or } x = \frac{\pi}{2k}$ $Area = \int_0^{\frac{\pi}{2k}} (\sin kx - \sin^2 kx)\mathrm{d}x$ $= \int_0^{\frac{\pi}{2k}} (\sin kx - \frac{1}{2} + \frac{1}{2}\cos 2kx)\mathrm{d}x$ $= \left[\frac{-1}{k}\cos kx - \frac{1}{2}x + \frac{1}{4k}\sin 2kx \right]_0^{\frac{\pi}{2k}}$ $= \left(\frac{-1}{k}\cos\frac{\pi}{2} - \frac{\pi}{4k} + \frac{1}{4k}\sin\pi \right) - \left(\frac{-1}{k}\cos 0 - 0 + \frac{1}{4k}\sin 0 \right)$ $= \frac{1}{k} - \frac{\pi}{4k}$ $= \frac{1}{k}\left(1 - \frac{\pi}{4}\right)$ $= \frac{4-\pi}{4k}$	M E	 Or equivalent.	

Judgement Statement

Achievement	Achievement with Merit	Achievement with Excellence
Integrate functions and use integrals to solve problems. $4 \times$ A including at least $1 \times$ A1 and $1 \times$ A2	Use advanced integration techniques to find integrals and solve problems. Achievement plus $2 \times$ M OR $3 \times$ M	Solve more complex integration problem(s). Achievement with Merit plus $1 \times$ E OR $2 \times$ E

The following Mathematics-specific marking conventions may also have been used when marking this paper:

- Errors are circled.
- Omissions are indicated by a caret (∧).
- **NS** may have been used when there was not sufficient evidence to award a grade.
- **CON** may have been used to indicate 'consistency' where an answer is obtained using a prior, but incorrect answer and **NC** if the answer is not consistent with wrong working.
- **CAO** is used when the 'correct answer only' is given and the assessment schedule indicates that more evidence was required.
- # may have been used when a correct answer is obtained but then further (unnecessary) working results in an incorrect final answer being offered.
- **RAWW** indicates right answer, wrong working.
- **R** for 'rounding error' and **PR** for 'premature rounding' resulting in a significant round-off error in the answer (if the question required evidence for rounding).
- **U** for incorrect or omitted units (if the question required evidence for units).
- **MEI** may have been used to indicate where a minor error has been made and ignored.